State space :

* Representation (Model) → Controllable form, Observable form, Diagonal form
* Analysis
* Design

Ex:

$$\frac{x_1}{x_2} = \frac{1}{s}$$
$$\dot{x}_1 = x_2$$

$$\frac{x_2}{x_3} = \frac{K_m}{Js + B}$$
$$Js\,x_2 + B x_2 = K_m x_3$$
$$J\dot{x}_2 + B x_2 = K_m x_3$$
$$\dot{x}_2 = \frac{K_m}{J} x_3 - \frac{B}{J} x_2$$

$$\cancel{L_a s x_3}$$
$$\frac{x_3}{V_a - K_b x_2} = \frac{1}{L_a s + R_a}$$
$$L_a s x_3 + R_a x_3 = V_a - K_b x_2$$
$$L_a \dot{x}_3 + R_a x_3 = V_a - K_b x_2$$
$$\dot{x}_3 = \frac{V_a}{L_a} - \frac{K_b}{L_a} x_2 - \frac{R_a}{L_a} x_3$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{B}{J} & \frac{K_m}{J} \\ 0 & -\frac{K_b}{L_a} & -\frac{R_a}{L_a} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L_a} \end{bmatrix} V_a$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

form

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

To make the system state space to be in the controllable form, then ~~if~~ any change in the i/p then the states should change. So the B matrix should be a non-zero element matrix $\boxed{2}$

special case from controllable form is the canonical controllable form

Observable

if the C matrix of the system is non zero element matrix then the form is said to be Observable form.

Diagonal form

$$A = \begin{bmatrix} \ddots & 0 \\ 0 & \ddots \end{bmatrix}$$

poles of the system

Diagonal form make the system is easy to analyse the stability

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Analysis of state space ..

$\dot{x} = Ax + Bu \xrightarrow{\text{and}} y = Cx$

$\Downarrow \mathcal{L}$

the system have Actual C.L poles locations

$sx = Ax + Bu$

$(sI - A)x = Bu$

$x = (sI - A)^{-1} Bu$

$y = Cx$

$= C(sI - A)^{-1} Bu$

$\therefore \frac{y}{u} = C(sI - A)^{-1} B$

$C/Cq : |sI - A| = 0$

* Design problem

$\dot{x} = Ax + Bu$ if $u = -kx$

$\dot{x} = Ax - BKx$

$Sx = (A - BK)x$

$(S - A + BK)x = 0$

$|SI - A + BK| = 0$

Actual ~~o.l.~~ poles
c/c eq $|SI - A| = 0$

if the desired c/c eq. $\alpha(s)$.

(this topic ~~is~~ will be detailed next section)